

Exercise 42

Evaluate the integral.

$$\int_{1/2}^{1/\sqrt{2}} \frac{4}{\sqrt{1-x^2}} dx$$

Solution

Bring the constant 4 in front of the integral.

$$\int_{1/2}^{1/\sqrt{2}} \frac{4}{\sqrt{1-x^2}} dx = 4 \int_{1/2}^{1/\sqrt{2}} \frac{1}{\sqrt{1-x^2}} dx$$

Recognize that the integrand is the derivative of the inverse sine function (page 214).

$$\int_{1/2}^{1/\sqrt{2}} \frac{4}{\sqrt{1-x^2}} dx = 4 \int_{1/2}^{1/\sqrt{2}} \frac{d}{dx}(\sin^{-1} x) dx$$

Use the second part of the fundamental theorem of calculus to evaluate the integral.

$$\begin{aligned} \int_{1/2}^{1/\sqrt{2}} \frac{4}{\sqrt{1-x^2}} dx &= 4(\sin^{-1} x) \Big|_{1/2}^{1/\sqrt{2}} \\ &= 4 \left(\sin^{-1} \frac{1}{\sqrt{2}} - \sin^{-1} \frac{1}{2} \right) \\ &= 4 \left(\frac{\pi}{4} - \frac{\pi}{6} \right) \\ &= 4 \left(\frac{\pi}{12} \right) \\ &= \frac{\pi}{3} \end{aligned}$$